Real Analysis MAT425 | Exam 1: In Class Part | February 28, 2008

Prof. Thistleton

1. Prove the following result using mathematical induction.

$$\frac{n(n+1)(2n+1)}{6} = \sum_{k=1}^{n} k^2$$

2. Denote the complement of a set A by $\overline{A}$. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

3. Find the lim inf and lim sup of the following sequence: $a_n = \frac{1}{n} + cos(n\pi)$

4. Let x be interior to both of sets V and W. Show that x is interior to $V \cap W$ and $V \cup W$

5. Calculate

$$\sum_{k=2}^{\infty} (\frac{3}{4})^k$$

6. Does the following series converge? Support your answer!

$$\sum_{k=0}^{\infty} \frac{k^2}{k!}$$

7. Does the following series converge? Support your answer!

$$\sum_{k=2}^{\infty} \frac{1}{klog(k)}$$

Real Analysis MAT425 Exam 1: Take Home Part due Wednesday March 6, 2008
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1. Suppose that a sequence (a_n) has a limit, a. Define a new sequence (s_n) as follows:

$$s_1 = a_1$$

$$s_k = \frac{a_1 + a_2 + \ldots a_k}{k}, \qquad k = 2, 3, \ldots$$

Show that

$$\lim_{n \to \infty} s_n = a$$

2. The following question is adapted from one that appeared on the Doctoral Qualifying Examination given at Stony Brook during the Spring 1995 semester.

   Let $a_0 = 1$. Define the sequence

   $$a_{n+1} = \frac{1}{2}a_n + \frac{3}{2a_n}$$

   and show that

   (a) Write out the first 4 terms of the sequence.

   (b) $a_n^2 \geq 3$ for n = 1, 2, ...

   (c) a_n is monotone decreasing.

   (d) a_n^2 is a Cauchy sequence.

   (e) Calculate $\lim_{n\to\infty} a_n$.