

1. Calculate the radius of convergence for the following series:

(a) $\sum_{n=1}^{\infty} 2^n x^n$

(b) $\sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n$

(c) $\sum_{n=1}^{\infty} \frac{n^2}{2^n} x^n$

(d) $\sum_{n=1}^{\infty} \frac{2^n}{n!} x^n$

2. Let $S = [-1, 1]$ and define $f : S \rightarrow \mathfrak{R}$ as

$$f(x) = \begin{cases} 0 & x \text{ rational} \\ x & x \text{ irrational} \end{cases}$$

Is f continuous anywhere?

3. Show that if $\sum a_n$ converges, so does $\sum a_n^2$.

4. Suppose $f'(x) < 0$ on (a, b) . Show that f is strictly decreasing.

5. Define $f(x) = |x|^3$ and calculate $f'(x)$ and $f''(x)$. Does $f'''(x) = f^{(3)}(x)$ exist for $x = 0$?

6. Find the limit function $f(x) = \sum_{n=2}^{\infty} n(n-1)x^{n-2}$ for $|x| < 1$.

7. Using mathematical induction, show that $2^n = \sum_{k=0}^n \binom{n}{k}$ where $\binom{n}{k} \equiv \frac{n!}{(n-k)!k!}$.

8. Let

$$f_n(x) \equiv \frac{x^2}{(1+x^2)^n}, \quad n = 1, 2, 3, \dots$$

on $S = (-\infty, \infty)$ and define $f(x) \equiv \sum_{n=1}^{\infty} f_n(x)$. Calculate the limit function $f(x)$. (Recall the formula for the geometric series.) Is the convergence uniform on $(-\infty, \infty)$?