Real Analysis MAT425 | Final | Wed., Apr. 30 8:00-10:00 a.m.
Prof. Thistleton

1. A sequence (a_n) converges to a limit $L > b$. Show that there esists N such that $n > N \Rightarrow a_n > b$.

2. Show that the union of two compact sets is also compact.

3. Define a sequence as follows: $s_1 \equiv 1$ and $s_{n+1} = \frac{1}{4}(s_n + 1)$.

(a) Calculate s_2, s_3, and s_4.

(b) Show that $s_n > \frac{1}{3}$ for all n using induction.

(c) Show that (s_n) is non-increasing, hence convergent.

(d) Calculate $\lim_{n\to\infty} s_n$.

4. Calculate

$$\lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2}\, dt$$

5. Define

$$f(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 \leq x \leq 1 \\ 1 & 1 < x \end{cases}$$

and define $F(x) = \int_0^x f(t)dt$.

(a) Sketch $F(x)$

(b) Where is $F(x)$ differentiable? Calculate $F'(x)$ where $F(x)$ differentiable.

6. Show whether $\sum_{n=2}^{\infty} \frac{log(n)}{n}$ converges or diverges.

7. Show whether $\sum_{n=2}^{\infty} \frac{log(n)}{n^2}$ converges or diverges.